A Novel Index for Online Voltage Stability Assessment Based on Correlation Characteristic of Voltage Profiles

M. R. Aghamohammadi*, S. S. Hashemi** and M. S. Ghazizadeh*

Abstract: Voltage instability is a major threat for security of power systems. Preserving voltage stability margin at a certain limit is a vital requirement for today’s power systems. Assessment of voltage stability margin is a challenging task demanding sophisticated indices. In this paper, for the purpose of on line voltage stability assessment a new index based on the correlation characteristic of network voltage profile is proposed. Voltage profile comprising all bus voltages contains the effect of network topology, load-generation patterns and reactive power compensation on the system behavior and voltage stability margin. Therefore, the proposed index is capable to clearly reveal the effect of all system characteristics and events on the voltage stability margin. The most attractive feature for this index is its fast and easy calculation from synchronously measured voltage profile without any need to system modeling and simulation and without any dependency on network size. At any instant of system operation by merely measuring network voltage profile and no further simulation calculation this index could be evaluated with respect to a specific reference profile. The results show that the behavior of this index with respect to the change in system security is independent of the selected reference profile. The simplicity and easy calculation make this index very suitable for online application. The proposed approach has been demonstrated on IEEE 39 bus test system with promising results showing its effectiveness and applicability.

Keywords: Voltage Stability Margin, Voltage Profile, Pattern Recognition, Correlation, Load Pattern.

1 Introduction
Economic and operational factors make power systems to utilize maximum percentage of their transmission capacity and consequently operate close to stability limit with fewer margins. In such environment voltage instability is emerged as a major threat for power system security. Nowadays most of electric utilities use fast response excitation systems, faster relays to reduce the fault clearing time and other control devices, so, the transient stability limitations, have become less restrictive. Power systems world-wide have become increasingly concerned with voltage stability and collapse problems [1]. Several major voltage collapse (VC) phenomena have been experienced by utilities in recent years. These VC phenomena usually result in widespread blackouts [2]. A number of these collapse phenomena were reported in France, Belgium, Sweden, Germany, Japan, and the United States [3, 4]. Voltage collapse is basically a dynamic phenomenon with rather slow dynamics in time domain from a few seconds to some minutes or more [5]. It is characterized by a slow variation in system operating point due to increase in the loads in such a way that the voltage magnitude gradually decreases until a sharp accelerated change occurs.

In spite of the dynamical nature of the voltage instability problem, a number of methodologies based on static approaches have been proposed to evaluate this problem. The adequacy of those static approaches for the analysis of voltage stability problem is supported by the fact that the system dynamics influencing voltage stability are usually slow [6-8] and if system models are chosen properly, the dynamical behavior of the power system may be closely approximated by a series of snapshots matching the system condition at various time frames along the time domain trajectory [6, 9]. Numerous research papers [10] have been devoted to the analysis of both static and dynamic aspects of voltage stability. Voltage stability has been emerged as a main limit for loading and power transfer capability of power systems. In order to preserve voltage stability margin at a desired level, on line assessment of security margin is highly demanded which is a challenging task.
Several works have been conducted previously for the prediction of voltage stability and proximity to collapse conditions based on the load flow calculation techniques, using sensitivity indices and continuation methods [9,11-13], singular value of Jacobian matrix [14,15], load flow feasibility [6,7]. Continuation power flow based voltage stability analysis techniques are fairly accurate and determine the voltage collapse point accurately and identify the critical buses, but all these methods are usually time consuming and not suitable for online applications. Some methods utilized the system Jacobian matrix [9,12-14,16] by exploiting either its sensitivity or its eigenvalue behavior to determine its vicinity to singularity. In [16] an enhanced method for estimating look-ahead load margin to voltage collapse, due to either saddle-node bifurcation or the limit-induced bifurcation, is proposed. In [1], a static approach based on both optimal load flow (OLF), conventional load flow (LF) solutions and singular value decomposition of the load flow Jacobian matrix (J) is proposed for assessing the steady-state loading margin to voltage collapse of the North-West Control Area (NWCA) of the Mexican Power System. In [17], derivative of apparent power against the admittance of load (dS/dY) is proposed for measuring proximity to voltage collapse. It may be easily accomplished, because both the power and the admittance are measurable, and the change of load occurs continuously, as a result of switching on and off the impedances, and/or actions of the transformer on load tap-changing devices. The techniques proposed in [2] are able to evaluate voltage stability status efficiently in both pre-contingency and post-contingency states with considering the effect of active and reactive power limits. The proposed method also employs the adaptive continuation technique and local analysis of the contingency effect. In [5], based on the fact that the line losses in the vicinity of voltage collapse increase faster than delivery of the apparent power and by using local phasors’ magnitudes and angles, a change in apparent power line flow in a time interval is exploited for computation of the voltage collapse criterion. In [18] by means of the singular value decomposition (SVD) of Jacobian matrix the MIMO transfer function of multi-machine power system for the analysis of the static voltage stability is developed. In this method, all possible active and reactive power control buses are considered as the input to the MIMO model. Moreover, the more critical buses affected by the static voltage stability are considered as the output of the MIMO model. In [19], operating variable information concerning the base system condition as well as the contingency itself like line flow, voltage magnitude and reactive reservation in the critical area are used to provide a complex index of the contingency severity. In [8], modal analysis and minimum singular value are used to analyze voltage stability and estimate the proximity of system condition to voltage collapse. Furthermore, the critical buses and the weakest branches in the transmission system are recognized. Some other indices are developed based on intelligent computation technique [16, 20]. In [16], a feed forward neural network is used to evaluate L index for all buses. In [20] for on line voltage stability assessment of each vulnerable load bus an individual feed forward type of ANN is trained. In this method, ANN is trained for each vulnerable load bus and for a wide range of loading patterns. In [21], using contingency screening and ranking techniques most critical contingencies are identified. In [22], a neural network-based approach for contingency ranking of voltage collapse is proposed. For this purpose by using the singular value decomposition method, a Radial Basis Function (RBF) neural network is trained to map the operating conditions of power systems to a voltage stability indicator and contingency severity indices corresponding to transmission lines.

Voltage stability indices could be basically categorized in two types as 1-model based indices and 2-non model based indices. Most voltage stability indices are model based which are evaluated from power system simulation results requiring system model, network data, load and generation. Evaluation of these indices is usually time consuming due to the process of simulation calculation. Non model based indices are evaluated from system operating variables without any need to simulation calculation and network modelling. The model based indices are not suit for on line assessment rather than non model based ones.

In this paper, a novel approach is presented for assessment of voltage stability margin using correlation index extracted from bus voltage profiles. This index is non model based one which only needs bus voltage profile that could be measured and gathered synchronously for all buses. Bus voltage profile of the network is relatively a complete feature of system behavior containing the effect of network structure, load-generation pattern and system controllers. The correlation of network voltage profile with respect to a reference profile provides valuable information about system voltage stability including the effect of network structure, load-generation patterns and other controllers.

2 Pattern Recognition

Pattern Recognition (PR) is an identification technique which is able to extract the dominating feature of a pattern for recognizing new patterns [23]. Pattern recognition has various applications in electric
power systems, such as classification of load profiles, recognition of waveforms and disturbances in power quality [23]. Given a pattern, its recognition/classification may consist of one of the following tasks:

- Supervised classification in which the input pattern is identified as a member of a predefined class.
- Unsupervised classification (e.g. clustering) in which the pattern is assigned to a hitherto unknown class.

Note that the recognition problem here is being used as a classification or categorization task, where the classes are either defined by the system designer (in supervised classification) or learned based on the similarity of patterns (unsupervised classification). Pattern recognition essentially involves the following three tasks:

- Data acquisition and preprocessing
- Data representation
- Decision making

The best known approaches for pattern recognition are as follows:

- Template matching
- Statistical classification
- Syntactic or structural matching
- Neural networks

These models are not necessarily independent and sometimes the same pattern recognition method exists with different interpretation. In this paper, the template matching approach based on the correlation distance is used for recognizing and extracting main feature of a given voltage profile.

3 Correlation of Network Voltage Profiles

In this paper, all bus voltage magnitudes which are measured and gathered synchronously at a specific operation instant constitute network voltage profile. For each instant of power system operation, there is a corresponding voltage profile which represents system operating condition including effects of network data and topology, load-generation patterns and all system controllers. Network voltage profile could be considered as a representative for system voltage stability and could be used as a measure for voltage stability margin. The trend of variation of voltage profile contains useful information about voltage stability and its proximity to stability limit. In the network voltage profile, bus voltages could be ordered based on a predefined bus numbering. Fig. 1 illustrates a series of network voltage profiles obtained for IEEE 39-bus test system corresponding to consequent increase of load level toward voltage collapse. The change in bus ordering may change the shape of voltage profile but it does not affect the correlation characteristic of the profile.

In this paper, it is shown that in the time trend of system operation, the correlation of network voltage profile associated with each operating instant and corresponding voltage profile could be taken as a system feature for voltage stability assessment and its proximity to voltage collapse. Correlation of the patterns is a general and robust technique for pattern recognition. Correlation recognition is enumerated as a subset of statistical pattern recognition [24].

Correlation factor between two voltage profiles $V_i$ and $V_j$ is denoted by $R_{ij}$ and evaluated using Eq. (1).

$$\begin{align*}
R_{ij} &= \frac{\sum V_i \cdot \sum V_j - \sum V_i \cdot \sum V_j}{\sqrt{\left(\sum V_i^2 - (\sum V_i)^2\right)\left(\sum V_j^2 - (\sum V_j)^2\right)}} \\
&= \frac{\sum V_i \cdot \sum V_j - \sum V_i \cdot \sum V_j}{\sqrt{\left(\sum V_i^2 - (\sum V_i)^2\right)\left(\sum V_j^2 - (\sum V_j)^2\right)}}
\end{align*}$$

where, $n$ is the number of bus voltages in each profile.

Correlation factor takes a value between zero and one. If no dependency and resemblance exist between two voltage profiles then the correlation coefficient will be zero and it means that the voltage profiles are totally uncorrelated and don’t have any common features. Correlation factor of two voltage profiles away from zero and close to one indicates more dependency, resemblance and common features between those profiles. Therefore, in an online environment, correlation factor of two voltage profiles could be evaluated very fast for assessing voltage stability associated to the operating condition corresponding to the given voltage profile.

4 Voltage stability Assessment by Correlation Index

Figure 2 illustrates bus voltage variation versus system load increment denoted as P-V curve. Voltage stability margin (VSM) is defined as the proximity of the operating point to the point of voltage collapse in terms of power difference evaluated by Eq. (2).

$$\text{VSM}_{\alpha,i} = P_{\text{max},i} - P_{\alpha,i}$$

(2)

where, $P_{\alpha,i}$ and $P_{\text{max},i}$ are system operating load and loadability limit under a given loading pattern $\alpha$ respectively. Loading pattern is a vector $\alpha$ indicating the

![Fig. 1 Network voltage profiles corresponding to consequent increase of load level toward voltage collapse.](image-url)
contribution of load buses during system load increment toward the point of voltage collapse. In equation (3) \( \alpha_k \) represents the share and contribution of bus \( k \) for load pick up in the process of system load increment [25].

\[
\alpha_k = \frac{P_{load,k}}{\sum P_{load,k}}
\]

Network structure, loading pattern and reactive power compensation are dominant factors affecting loadability limit and voltage stability margin. For a given network structure and reactive power compensation, by increasing system load according to a specific loading pattern, the system will reach to a specific loadability limit. Therefore, each loading pattern can be characterized by an associated loadability limit which dictate system security margin.

In the trend of system movement toward voltage stability limit, the network may experience different voltage profiles corresponding to different load levels. Each voltage profile contains the effect of network structure and its data, load-generation patterns, reactive power compensation and other controllers. Also, it contains useful information about stability status and voltage stability margin of the system. For each voltage profile there are two associated features, 1-system operating load level (Po) and 2- system loadability limit (Pmax), from which voltage stability margin VSM could be evaluated by Eq. (2). By changing network structure, reactive power compensation and loading pattern, system voltage profiles will also change in a way that reflect the change in voltage stability margin.

The main objective of this paper is to derive an on line security index which could be evaluable by merely synchronously measured network voltage profiles. For this purpose, correlation factor between network voltage profiles is adopted as on line voltage stability index. Larger value for correlation factor indicates solidarity and reinforcement in system from view point of voltage stability, while lower value indicates weak stability. In order to evaluate correlation factor associated with each voltage profile a reference voltage profile is required. The choice of this reference profile does not affect the trend of change in correlation index with respect to change in system security margin. However, it is better to select the reference profile from a loading pattern with medium loadability margin and at a base load.

In this approach, as the power system moves with a dynamic or quasi steady state behavior along its trajectory, at each snapshot, system voltage profile could be measured and its correlation factor can be evaluated on line as voltage stability index without any need to system modelling and simulation calculation. Fig. 3 shows the trend of the correlation change of voltage profiles for IEEE 39-bus test system as the system moves toward stability limit according to different loading patterns. Network structure and reactive power compensations are remained constant.

As it can be seen as long as the load increment is the only cause for system movement toward stability limit, with no change in network structure or reactive power, the trend of correlation change is uniform and smooth with a relatively fixed descend rate. However, voltage profiles belonging to the same operating load level but with different loadability limit have different correlation factors which show different security margins. As it will be seen in the next section, following a sudden change in the network structure or reactive power compensation, correlation factor bear a sudden increase or decrease indicating a sudden change in the system security margin.

5 Simulation Studies and Results

In order to show the effectiveness of the proposed index, it is demonstrated on the IEEE 39-bus New England system shown in Fig. 4.

In order to prepare several voltage profiles with different security margin, 26 loading patterns with different associated loadability limits in the range of 7000 to 12800 MW are defined. For each loading
pattern, system load is increased incrementally by step of 5% until the point of voltage stability limit resulting in individual loadability limit. With respect to each specific loading pattern, in the trend of load increment and system movement toward voltage stability limit a certain number of voltage profiles are created by load flow calculation. Each voltage profile belonging to a specific loading pattern and a specific operating load level is associated with a certain value of voltage stability margin. In order to investigate the effect of network structure and reactive power on the voltage profile and its associated security margin, for each loading pattern some lines or reactive power sources are taken out as single contingency.

In this study, by three scenarios the effect of load increment, line outage and reactive power changes on the voltage stability margin are investigated and also the ability of the proposed index for addressing voltage stability margin is demonstrated. In order to evaluate the correlation factor of voltage profiles, two voltage profiles with the same load level 6096 MW but belonging to different loading patterns are adopted as reference profile.

1. Voltage profile belonging to loading pattern with loadability limits 10059 MW
2. Voltage profile belonging to loading pattern with loadability limits 11278 MW

### 5.1 Effect of Change in Loading Pattern

In this scenario, system load started to increase from load level 6096 MW according to the loading pattern corresponding to loadability limit Pmax=10059 MW. Generation pattern is kept fixed. Figs. 5 and 6 show correlation variation of system voltage profiles.

Figure 5 shows the variation in the correlation of voltage profiles due to load increment and change in loading pattern. This case all correlations are evaluated with respect to the reference profile 1. Comparing Figs. 5 and 6, it can be concluded that regardless of the choice of reference profile, the behaviour of correlation index with respect to load increment or change in loading pattern are the same.

### Table 1 Change in correlation factor and VSM due to change in loading pattern.

<table>
<thead>
<tr>
<th>Step</th>
<th>Loading Pattern</th>
<th>Load</th>
<th>Pmax</th>
<th>VSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>10059 MW</td>
<td>6096</td>
<td>10059</td>
<td>3353</td>
</tr>
<tr>
<td>2nd</td>
<td>10059 MW</td>
<td>6706</td>
<td>10059</td>
<td>3049</td>
</tr>
<tr>
<td>3rd</td>
<td>10059 MW</td>
<td>7010</td>
<td>8535</td>
<td>1525</td>
</tr>
<tr>
<td>4th</td>
<td>10059 MW</td>
<td>7010</td>
<td>12802</td>
<td>5792</td>
</tr>
</tbody>
</table>

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**Fig. 5** Change in correlations factor due to load increment and sudden change in loading pattern (reference= profile 1).
initial load level 6096 MW according to the loading pattern corresponding to loadability limit 10059 MW. Fig. 9 shows correlation variation of voltage profiles due to load increment and reactive power compensation evaluated with respect to the reference profile 1. After two steps of load increment, at load level 7010MW a 200 Mvar inductive or capacitive reactive power is added at some buses. As it can be seen by inductive reactive power compensation, correlation factor suddenly dropped showing a sudden decrease in voltage stability margin. In fact line outage and reactive power reduce loadability limit and voltage stability margin. The reduction caused in security margin due to reactive power compensation is different from bus to bus.

### 5.2 Effect of Line Outage

In this section, the effect of line outage on the correlation index of voltage profiles is investigated. In this scenario, system load started to increase from the initial load level 6096 MW according to the loading pattern corresponding to loadability limit 10059 MW. Fig. 7 shows correlation variation of voltage profiles due to load increment and also line outage which is evaluated with respect to reference profile 1. At the third step of load increment with load level 7010MW a line outage happened. As it can be seen due to line outage, correlation factor suddenly dropped showing a sudden decrease in voltage stability margin. In fact line outage weakens network structure which results in smaller loadability limit and consequent reduction in voltage stability margin. The reduction caused in security margin due to a line outage is different from bus to bus.

Figure 8 shows the change in correlation of voltage profile due to different line outages at the 3rd step. Comparing the outage of line 4-5 with line 5-6, it can be seen the outage of line 4-5 which caused more reduction in security margin has resulted in lower correlation.

### 5.3 Effect of Reactive Power Change

In this section, the effect of change in reactive power on the correlation index of voltage profiles is investigated. In this scenario, system load started to increase from the initial load level 6096 MW according to the loading pattern corresponding to loadability limit 10059 MW.
Table 3 shows the change caused in correlation of voltage profile due to reactive power compensation at buses 3, 8 and 16 at the 3rd step. In Fig. 9 it can be seen that compensation at buses 3 and 16 which caused more reduction in voltage stability margin compared to bus 8, it has also resulted in more reduction in correlation.

Figure 10 shows change in correlation of voltage profiles due to load increment and reactive power compensation for the same scenario as Fig. 9 but with respect to the reference profile 2.

Table 4 shows correlation index and VSM for each case. In both scenarios, the highest correlation is achieved when reactive power is compensated at bus 8.

5.4 Effect of Combinatorial Events

In this section, the ability of the proposed index for evaluating system voltage stability margin in the case of occurrence a combinatorial event is demonstrated. For this purpose, the effect of combinatorial events on voltage correlation and corresponding VSM is investigated.

A combinatorial event consists of two events like a simultaneous events consisting a line outage and change in loading pattern to weaker case, 2-change of loading pattern to a stronger case, 3-change of loading pattern to a weak case accompanied with a line outage and 4-change of loading pattern to initial pattern with reclosing line outage, are examined.

Scenario 1: Figs. 11 and 12 show variation in correlation index evaluated with respect to reference profiles 1 and 2 respectively. Correlation indices associated with 4 events are indicated by 4 points respectively. Point (1) and point (2) indicate correlation corresponding to change of loading pattern to ones with Pmax=8535 MW and Pmax=12802 MW respectively. Point (3) indicates correlation after occurrence of simultaneous combinatorial events consisting a line outage and change in loading pattern to Pmax=9145 MW. Finally, point (4) indicates correlation corresponding to situation in which outage line has reclosed and loading pattern changed to initial pattern with Pmax=11059 MW.

Scenario 2: Figs. 13 and 14 show variation in correlation index evaluated with respect to reference profiles 1 and 2 respectively. Point (1) indicate correlation index corresponding to change of loading pattern to one with Pmax=9145MW. Point(2) indicates correlation after occurrence of simultaneous combinatorial events consisting a line outage and change in loading pattern to Pmax=8840 MW. Point (3) indicate correlation corresponding to change of loading pattern to one with Pmax=12802 MW. Finally, point (4) represents the situation in which outage line has reclosed and loading pattern has returned to initial pattern with Pmax=10059 MW.

Table 4 shows correlation index and VSM for different points during scenario 1 as shown in Fig. 12. Point (2)\' shows the situation a prior to occurrence of combinatorial even and point (3)\' refers to situation if combinatorial even did not occur.
Considering results the following points can be concluded:
1. As long as the change in VSM is due to only daily load increment based on a specific loading pattern, the correlation index decreases smoothly showing a smooth change in VSM.
2. Any sudden change due to either single event like line outage, reactive power outage, change in loading pattern or combinatorial events, imposes a sudden change in correlation index indicating a corresponding sudden change in VSM.
3. Combinatorial events can impose worse security with less VSM which is clearly reflected by sharp change of correlation index. In other words, any change in network topology or system conditions resulted by either single or simultaneous combinatorial events consisting of two or more events which cause a sharp change in VSM can be clearly reflected by a sharp change in correlation index.
4. Therefore, departing from type and number of events consisting a combinatorial event which cause major change in system condition and security, correlation index is able to clearly explore system VSM.

6 Conclusion
In this paper, a new index based on the correlation of network voltage profile has been proposed for online voltage stability assessment. The most interesting feature of this index is its easy calculation by using merely bus voltage profile which can be gathered by synchronous measurement of bus voltages at any instant of system operation without any need to system modeling and simulation calculation. This index takes a value between 0 and 1, such that a value closer to 1 points out voltage stability reinforcement and corresponding higher voltage stability margin, while a
small value indicates a lower voltage stability margin. The network voltage profile is an exhaustive representative of system operating condition which includes the effect of network topology (line outage), load level, loading pattern, generation pattern and reactive power compensation. For this reason, the correlation of voltage profile has shown its ability to reveal the effect of all abovementioned parameters on voltage stability margin. Also it is shown that regardless of which voltage profile is adopted as reference profile for correlation evaluation, the value of correlation factor of voltage profile is capable to reveal voltage stability margin and its change in the case of change in the network topology, loading pattern and reactive power compensation. These features prove the effectiveness and suitability of the proposed index for evaluating system voltage stability margin in conditions of happening different events affecting system security margin.

References


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